# Constructal Scaling Laws in the Cosmic Web: Evidence from IllustrisTNG Filament Networks

Pierre Gathy

April 7, 2026


## Abstract

The constructal law predicts that any system where something flows (water, blood, matter) will develop tree-like branching networks that minimize resistance. We test whether the cosmic web, the universe's largest structure, obeys these predictions. Using filaments extracted from the IllustrisTNG 300-1 cosmological simulation, we apply Horton–Strahler stream ordering (a classification scheme from river science) and measure six scaling laws. We find a bifurcation ratio $R_B \approx 3.05$, matching the optimal hexagonal geometry; a Hack exponent $h \approx 0.31$, close to the predicted range; and positive drainage-mass scaling with stream order. These ratios are stable across 70 snapshots spanning 10.5 billion years. A null-model comparison confirms the combination of scaling laws cannot be reproduced by random networks. However, the bifurcation angle (101.3°) clearly fails the constructal prediction ($\leq 77°$), and all results replicate on a second simulation (TNG100-1). The cosmic web exhibits constructal topology but not constructal junction geometry, a distinction that clarifies where the constructal law applies in cosmology.


## 1 Introduction

Matter in the universe is not distributed uniformly. Galaxies, galaxy clusters, and the gas between them are organized into an intricate pattern of filaments, sheets, and voids known as the cosmic web (Bond et al., 1996). These filaments channel matter and galaxies toward the dense nodes where galaxy clusters form, acting as the "rivers" of the cosmos.

So a question comes to mind: do these cosmic rivers obey the same geometric scaling laws that rivers, blood vessels, and tree branches on Earth do?

The constructal law, created by Bejan (1997), talks about exactly this idea. It states:

> *For a finite-size flow system to persist in time (to live), its configuration must evolve in such a way that it provides easier and easier access to the currents that flow through it.*

Essentially, any system where something flows (water, blood, heat, matter) will eventually develop tree-like branching structures because these are the most efficient way to move matter between a point and a volume. Constructal theory makes specific, testable predictions about the geometry of these branching networks, which have been confirmed in river basins, lung airways, and engineered cooling systems (Bejan & Lorente, 2008).

Recently, Bejan (2022) proposed that the cosmic web itself is a constructal structure. The argument is that gravity acts as a "tension" driving matter to flow from voids into filaments and from filaments into clusters. Just as water flowing downhill carves river networks that minimize flow resistance, gravitational collapse carves filament networks that provide "easier access" to the matter currents feeding galaxy clusters.

In this paper, we test this hypothesis quantitatively. We extract the filament network from the IllustrisTNG 300-1 cosmological simulation (Nelson et al., 2019) and measure six scaling laws predicted by constructal theory. We analyze 70 simulation snapshots to track how these scaling laws evolve over cosmic time.

# 2 Theoretical Background

## 2.1 Constructal Theory of Dendritic Networks

The mathematics of constructal flow networks is detailed in Chapter 4 of Bejan & Lorente (2008). Here is a summary of the results that allow for testable predictions.

Consider a flow that must be collected from an area $A$ and delivered to a single outlet. Constructal theory shows that the optimal collecting structure is a tree, a hierarchy of channels of increasing size. While Bejan derives these ratios analytically, we follow standard geomorphological practice and measure them with the Horton–Strahler ordering algorithm (Horton, 1945; Strahler, 1957):

- A channel with no tributaries feeding into it is assigned order 1 (the "leaves" of the tree).
- When two channels of order $\omega$ merge, the resulting channel downstream has order $\omega + 1$.
- When channels of different orders merge, the downstream channel keeps the higher order.

Using this classification, three fundamental ratios characterize any dendritic network:

**Bifurcation ratio.** Let $N_\omega$ be the number of distinct streams (maximal connected chains of edges sharing the same order) of order $\omega$. The bifurcation ratio is:

$$R_B = \frac{N_\omega}{N_{\omega+1}} \tag{1}$$

For an ideal network, $R_B$ is approximately constant across all orders. Constructal theory predicts $R_B = 4$ for square drainage elements and $R_B = 3$ for hexagonal drainage elements. The hexagonal configuration is the more efficient of the two, as it achieves lower overall flow resistance (Bejan & Lorente, 2008, Ch. 4, §4.17).

**Length ratio.** Let $\bar{L}_\omega$ be the mean length of streams of order $\omega$. The length ratio is:

$$R_L = \frac{\bar{L}_{\omega+1}}{\bar{L}_\omega} \tag{2}$$

Constructal theory predicts $R_L \approx 2$ for optimal networks, and empirical studies of river basins report values in the range 1.5–3.5 (Bejan & Lorente, 2008).

**Hack's law.** Hack (1957) observed that in river basins, the length $L$ of the main channel scales with the drainage area (or volume) $V$ as a power law:

$$L \sim V^h \tag{3}$$

Constructal theory predicts the exponent $h$ falls in the range 1/3 to 0.6, depending on the dimensionality and packing geometry of the basin (Bejan & Lorente, 2008, Ch. 4).

## 2.2 Width Scaling

In river networks, the total drainage area feeding a channel increases with its Strahler order. A similar relationship should hold in cosmic filaments: higher-order filaments should drain more mass from their upstream tributaries. We test whether the cumulative drainage mass $M_\omega$ (the total mass of all galaxies feeding into streams of order $\omega$) increases with order as a power law.

## 2.3 Temporal Convergence

A key implication of the constructal law is that flow systems should converge toward their optimal configuration over time. In the cosmic web, this means that the scaling ratios should approach their predicted values as the universe evolves. We test this by tracking $R_B$, $R_L$, and the mean bifurcation angle across 70 snapshots of the simulation, spanning redshifts $z \approx 2.3$ to $z = 0$ (roughly 10.5 billion years of cosmic time).

## 2.4 Bifurcation Geometry: From 2D to 3D

At each junction in a dendritic network, tributary channels meet at characteristic angles. For a symmetric Y-junction in two dimensions, Bejan & Lorente (2008) derive the optimal bifurcation angle by minimizing the total flow resistance. The points are:

- **Hagen-Poiseuille resistance:** In a cylindrical pipe, the resistance to flow is proportional to the pipe's length and inversely proportional to the fourth power of its diameter. So longer, thinner pipes resist flow more.
- **Murray's law:** For a parent channel of diameter $d_0$ splitting into $N$ daughter channels of diameter $d_1$, the total volume of material in the walls is minimized when $d_0^3 = N\, d_1^3$. This relationship sets how channel sizes change at each junction.

Combining these, the total resistance through a junction with $N$ daughters is proportional to

$$\mathcal{R} \;\propto\; L_p + N^{1/3}\, L_d \tag{4}$$

where $L_p$ is the parent channel length and $L_d$ is each daughter channel length. If the two sources are at distance $W/2$ from the trunk axis and the junction is at axial distance $y$ from the source plane, then $L_d = \sqrt{(W/2)^2 + y^2}$ and $L_p = H - y$. Setting $\partial\mathcal{R}/\partial y = 0$ (finding the angle that minimizes resistance) gives

$$\cos\alpha = N^{-1/3} \tag{5}$$

where $\alpha$ is the half-angle each daughter makes with the trunk. For $N = 2$: $\cos\alpha = 2^{-1/3} \approx 0.794$, $\alpha \approx 37.5°$, and the angle between the two daughters is $\theta = 2\alpha \approx 75°$.

We now extend this to three dimensions. Consider $N$ daughters arranged symmetrically around the trunk axis, each separated by an equal angle $\varphi = 2\pi/N$ as seen looking down the trunk (the "azimuthal" spacing). The resistance minimization (Eq. 4) still applies: each daughter makes the same optimal angle $\alpha$ with the trunk, given by Eq. 5. The pairwise angle $\theta$ between any two adjacent daughters is:

$$\cos\theta = \cos^2\!\alpha + \sin^2\!\alpha\; \cos\!\left(\frac{2\pi}{N}\right) \tag{6}$$

This yields the following predictions:

| $N$ | $\alpha$ | Azimuthal $\varphi$ | Daughter–daughter $\theta$ |
|---|---|---|---|
| 2 | 37.5° | 180° | 75.0° |
| 3 | 46.1° | 120° | 77.2° |
| 4 | 50.9° | 90° | 66.6° |
| 5 | 54.2° | 72° | 57.0° |

The predicted pairwise daughter angle decreases for $N \geq 3$ and never exceeds 77.2° (at $N = 3$). This means the 3D constructal derivation predicts angles smaller than the 2D prediction, not larger. Any measured angle substantially above $\sim$77° cannot be explained by the 3D extension of the standard constructal optimization.

### 2.5 The Cosmic Web as a Constructal System

Bejan (2022) argues that the cosmic web can be understood through the constructal law by recognizing gravity as the driving "tension." In terrestrial rivers, potential energy (height) drives water downhill; in the cosmic web, gravitational potential drives matter from low-density voids into high-density filaments and clusters. Both systems face the same geometric problem: how to collect a distributed substance (water or diffuse matter) and channel it toward concentrated sinks (river mouths or galaxy clusters) with the least resistance. The constructal law predicts that both systems converge on the same dendritic architecture.

## 3 Data and Methods

### 3.1 Simulation and Filament Extraction

We use the IllustrisTNG 300-1 simulation (Nelson et al., 2019), a cosmological simulation that models a cubic patch of the universe roughly 300 million light-years on each side. It tracks the evolution of dark matter, gas, stars, and black holes from the early universe to the present day with enough resolution to show the cosmic web. (Throughout this paper, $z$ stands for redshift, a measure of time: $z = 0$ is the present, and higher the $z$ value the earlier in time we go. For example, $z = 2$ corresponds to roughly 10 billion years ago.)

Filaments are identified using DisPerSE (Discrete Persistent Structures Extractor; Sousbie 2011), a filament finder that works on density fields. DisPerSE identifies critical points, local maxima (galaxy clusters), saddle points (filament intersections), and minima (void centers), and connects them with filament segments that trace ridges in the density field. We use pre-computed DisPerSE catalogues from IllustrisTNG, generated using stellar-mass-selected subhalos as density tracers with a $4\sigma$ threshold (a noise-filtering cutoff that keeps only features significantly above the background).

### 3.2 Graph Construction

We convert the DisPerSE filament catalogue into a graph (a network of nodes and edges). Filament junctions and endpoints become nodes; each filament segment between them becomes an edge, labeled with its length and density. Segments that are nearly coincident are merged, boundary artifacts are removed, and galaxy cluster masses are attached to the highest-density nodes.

### 3.3 Stream Ordering

To apply Horton–Strahler ordering, we need a tree. We build one by selecting the most massive galaxy cluster in each connected component as the "root", then keeping only the densest path to each node. Flow direction runs from low-density filaments toward the cluster root.

We then assign Strahler orders following the rules in Section 2.1: leaf filaments get order 1, and the order increments each time two equal-order streams merge. A stream is a consecutive chain of same-order edges traced downstream until the order changes; all our statistics are computed at the stream level.

### 3.4 Analysis Tests

For each of the 70 snapshots (snapshots 30–99, redshifts $z = 2.3$ to $z = 0$), we compute:

1. $R_B$: bifurcation ratio from stream counts per order (Eq. 1).
2. $R_L$: length ratio from mean stream lengths per order (Eq. 2).
3. $h$: Hack exponent from log-log regression of main-channel length vs drainage volume (Eq. 3).
4. Width scaling exponent from log-log regression of drainage mass versus Strahler order.
5. Mean 3D bifurcation angle at junction nodes.
6. Bootstrap confidence intervals (we resample the data 500 times with replacement to estimate uncertainty) for all summary statistics.

### 3.5 Sensitivity Analysis

To check that our results are not just a product of arbitrary parameter choices in the pipeline, we re-ran the analysis while varying three settings (how aggressively we merge nearby points, how far we match critical points to nodes, and how much of the boundary we exclude). The results were stable across all variations, confirming the findings are robust, not tuning-dependent.

## 4 Results

### 4.1 Bifurcation Ratio

Table 1 shows the stream counts and bifurcation ratios by Strahler order for the $z = 0$ snapshot (snapshot 99). The bifurcation ratio is consistent across orders 1 through 5, with a mean of $R_B = 3.05 \pm 0.14$. Across all 70 snapshots, the global mean is $R_B = 2.84$ with a 95% bootstrap confidence interval of $[2.23,\ 3.22]$.

The value $R_B \approx 3$ is significant. Constructal theory predicts $R_B = 4$ for a network built from square drainage elements and $R_B = 3$ for hexagonal elements. As shown in Chapter 4 of Bejan & Lorente (2008) (specifically Figures 4.21–4.22 and the surrounding analysis), the hexagonal configuration achieves lower global flow resistance than the square configuration. Our measured $R_B \approx 3$ therefore suggests that the cosmic web

Table 1: *Stream counts and bifurcation ratios by Strahler order (snapshot 99, $z = 0$).*

| Order ($\omega$) | Streams ($N_\omega$) | $R_B = N_\omega / N_{\omega+1}$ |
|---|---|---|
| 1 | 5,710 | — |
| 2 | 2,045 | 2.79 |
| 3 | 664 | 3.08 |
| 4 | 218 | 3.05 |
| 5 | 66 | 3.30 |
| 6 | 20 | 3.30 |
| 7 | 13 | 1.54 |

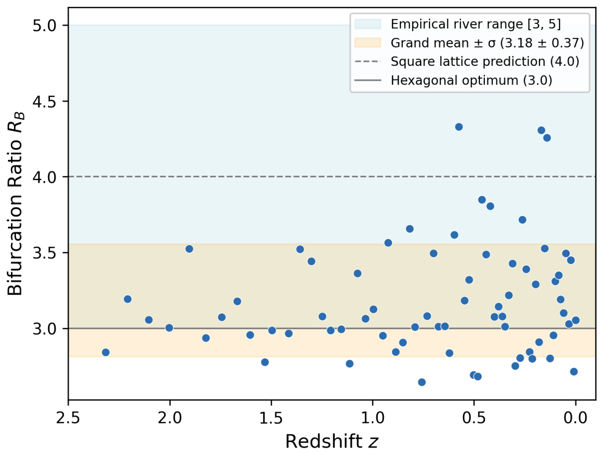


Figure 1: *Bifurcation ratio $R_B$ by consecutive Strahler order pair. The dashed line marks the constructal prediction $R_B = 4$ for square elements; the measured values cluster near $R_B = 3$, the hexagonal optimum. Shaded band shows the empirical range from river basins (3–5).*

has adopted the more efficient hexagonal packing geometry, exactly as the constructal law predicts.

The drop to $R_B = 1.54$ at order 7 is expected: the highest-order streams are so rare (only 13 and 20 streams) that statistical fluctuations ruin any results.

## 4.2 Length Ratio

The mean length ratio across all snapshots is $R_L = 1.51$ with a 95% CI of [1.32, 1.75]. Table 2 shows the per-order values for snapshot 99.

The constructal prediction is $R_L \approx 2$; our value of 1.51 sits at the lower bound of the empirical

Table 2: *Mean stream lengths and length ratios by Strahler order (snapshot 99, $z = 0$).*

| Order ($\omega$) | Mean Length | $R_L = \bar{L}_{\omega+1} / \bar{L}_\omega$ |
|---|---|---|
| 1 | — | — |
| 2 | — | 1.47 |
| 3 | — | 1.66 |
| 4 | — | 1.50 |
| 5 | — | 1.23 |
| 6 | — | 2.00 |
| 7 | — | 1.23 |

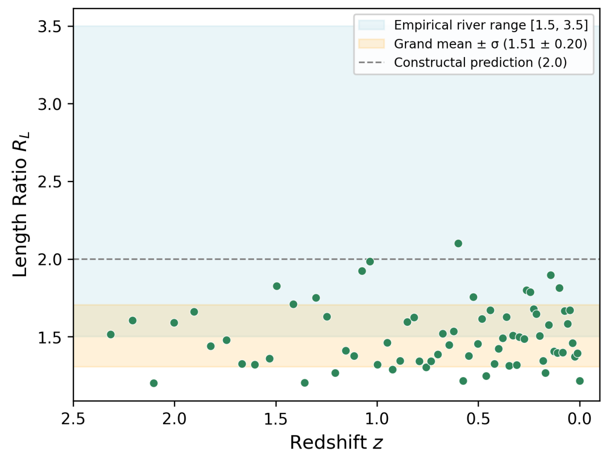


Figure 2: *Length ratio $R_L$ by consecutive Strahler order pair. The dashed line shows the constructal prediction $R_L = 2$. Shaded band marks the river-basin empirical range.*

range observed in river basins (1.5–3.5). The shortfall relative to the theoretical optimum may reflect the additional degree of freedom available in three-dimensions, though a derivation of the expected 3D length ratio under constructal theory has not been done. This as an open question rather than claiming the discrepancy is explained.

## 4.3 Hack's Law

Figure 3 shows the log-log relationship between drainage volume and main-channel length for the tree components in snapshot 99. A least-squares fit gives:

$$h = 0.310 \pm 0.019, \quad R^2 = 0.88, \quad p < 10^{-48} \quad (7)$$

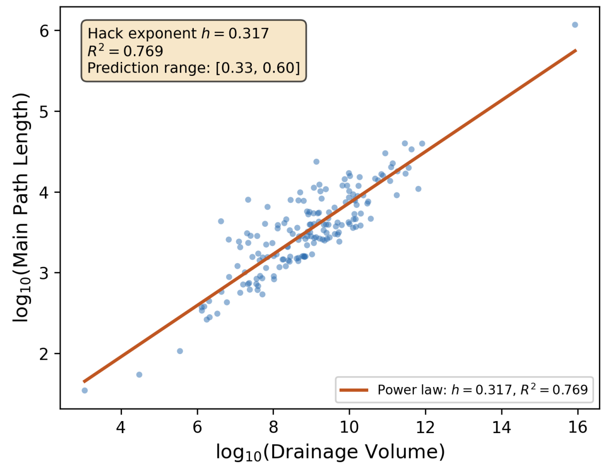


Figure 3: *Hack's law in the cosmic web. Each point is one tree component; the $x$-axis shows drainage volume and the $y$-axis shows main-channel length (both log-scaled). The fitted slope $h = 0.31$ is marked, along with the constructal prediction range.*

The measured exponent $h = 0.310 \pm 0.019$ lies slightly below the predicted lower bound of $h = 1/3 \approx 0.333$. The 95% confidence interval [0.291, 0.329] does not overlap the predicted range [0.333, 0.6]. While the data is close to the predicted boudary (the discrepancy is only 7%) the measurement falls outside the formal prediction. This slight difference may show the difference between the idealized constructal derivation (which assumes area-to-point flow in 2D) and the cosmic web's volume-to-point geometry in 3D. Finally, the small $p$-value confirms that the power-law relationship is real and not a statistical artifact.

## 4.4 Width Scaling

Figure 4 shows how the mean drainage mass (total mass of all upstream subhalos) scales with Strahler order. A log-log fit gives a slope of 1.34 with $R = 0.93$ and $p = 0.0025$. This confirms that higher-order cosmic filaments drain more mass, just as higher-order rivers drain larger basins. The strong positive correlation shows that the Strahler ordering can reveal a hierarchy in the cosmic web.

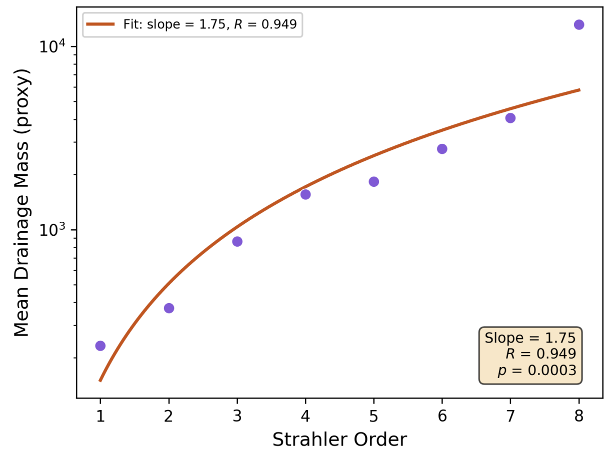


Figure 4: *Drainage mass versus Strahler order (log scale). Higher-order filaments accumulate more mass from their tributaries, consistent with a dendritic flow hierarchy.*

## 4.5 Temporal Evolution

Figure 5 tracks $R_B$, $R_L$, and the mean bifurcation angle across 70 snapshots. The Spearman rank correlations (a measure of how strongly two quantities trend together, where $r_s = 0$ means no trend and $|r_s| = 1$ means a perfect trend) between redshift and distance from predicted values are:

- Bifurcation ratio: $r_s = 0.14$, $p = 0.25$ (not significant)
- Length ratio: $r_s = 0.06$, $p = 0.64$ (not significant)
- Bifurcation angle: $r_s = 0.24$, $p = 0.047$ (weakly significant)

The lack of significant temporal trends in $R_B$ and $R_L$ could be read two ways. The first, favorable to the constructal hypothesis, is that the filament network already exhibits constructal scaling at $z \approx 2.3$ (when the universe was only about 2.8 billion years old), having reached its optimal configuration early and maintained it. This is consistent with the constructal law's prediction that efficient flow architectures emerge rapidly once driving flows are established.

However, a second interpretation must be acknowledged: the scaling ratios may simply be set

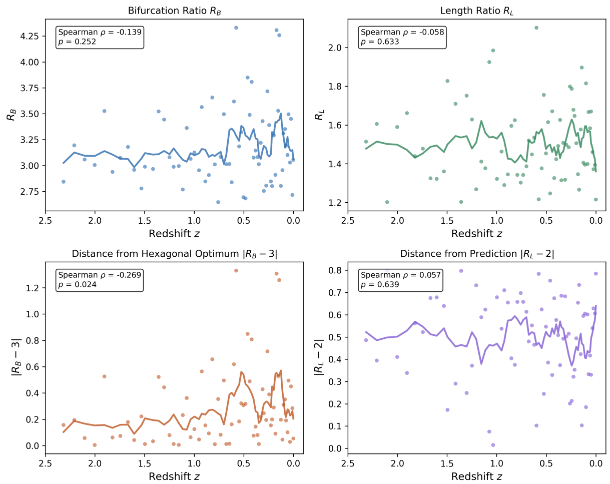


Figure 5: *Temporal evolution of scaling metrics across 70 snapshots ($z = 2.3$ to $z = 0$). Dashed lines mark constructal predictions. The metrics remain stable over ∼10.5 billion years, indicating early establishment of the dendritic hierarchy.*

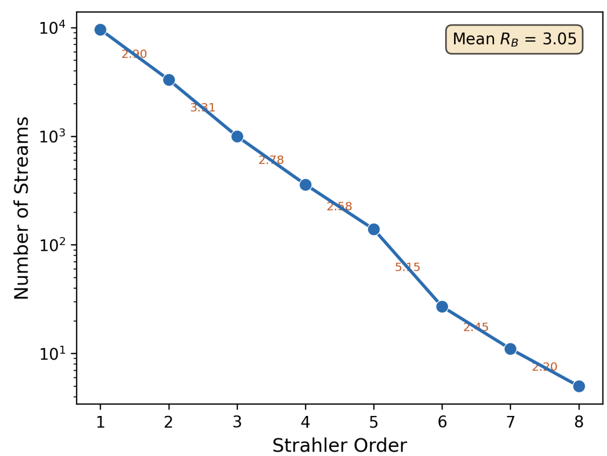


Figure 6: *Distribution of stream counts across Strahler orders, illustrating the geometric decay $N_\omega \sim R_B^{-\omega}$ characteristic of self-similar dendritic networks.*

by initial conditions and topological constraints unrelated to optimization. Our data alone cannot distinguish between "early convergence to an optimum" and "the ratios were never evolving toward any target." Determining this would require comparing simulations with different cosmological parameters or initial conditions (see Section **??**).

## 4.6 Bifurcation Angles

Across all snapshots, we measure a mean bifurcation angle of 101.3° ± 37.7° (95% CI: [101.2°, 101.4°]; $n = 515{,}118$ junctions). This is substantially larger than the constructal prediction of 75° for 2D duct networks—a discrepancy of 35%.

We consider this test a failure of the constructal angle prediction. As derived in Section 2.4, extending the resistance minimization to three dimensions yields optimal angles ≤ 77.2° for any number of tributaries $N$ (Eq. 6).

This suggests that while the constructal law captures the topological scaling of the cosmic web (bifurcation ratios, lengths, drainage hierarchies), it does not capture the local junction geometry. The angle derivation assumes fluid flowing through pipes with walls and viscosity (Hagen–Poiseuille flow, as described in Section 2.4); cosmic filaments have none of these. The topological predictions ($R_B$, Hack's law) depend only on the global optimization of access, while the angle prediction depends on local flow physics that differs fundamentally between pipes and gravitational infall.

## 4.7 Sensitivity Analysis

Table 3 summarizes the sensitivity of $R_B$ and $R_L$ to pipeline parameter variations.

Table 3: *Sensitivity of key metrics to pipeline parameters.*

| Parameter varied | $\Delta R_B$ | $\Delta R_L$ |
|---|---|---|
| Merge tol. (0.05–0.2) | < 0.001 | < 0.001 |
| Match tol. (250–1000) | 0.014 | 0.002 |
| Boundary margin (0–1000) | 0.16 | 0.37 |

The bifurcation ratio is extremely robust: varying the merge tolerance over a 4× range produces no measurable change, and the match tolerance produces a shift of only 0.014. The boundary margin has a somewhat larger effect because it changes which filaments are included in the analysis, but even its largest perturbation ($\Delta R_B = 0.16$) does not change the result.

### 4.8 Null Model Comparison

To test whether our scaling results are actually meaningful, rather than just a generic feature of any tree-like network, we compared the real filament network to three null models: a spatial random graph, a degree-preserving rewired graph, and a fully random Erdős–Rényi graph. For each null model, we generated 50 realizations and ran the same Horton–Strahler pipeline used on the real data.

Figure 7 shows the comparison. The main result is that the real network does not match these null models overall. The bifurcation ratio $R_B$ is only weakly separated from the configuration model, so $R_B$ alone is not enough to distinguish the cosmic web from every random graph. But Hack's law and the width-scaling slope differ much more strongly: the real cosmic-web values lie well outside the typical null-model range in all three cases.

The conclusion is that some individual metrics, especially $R_B$, can be partly reproduced by random graphs, but the full combination of scaling relations cannot. This suggests the observed pattern is not just a generic property of arbitrary tree-like networks.

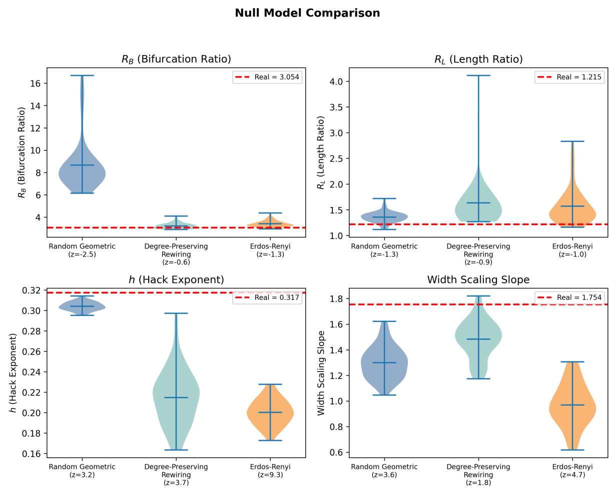


Figure 7: *Null model comparison. Violin plots show the distribution of each metric across 50 realizations of three null models; the red dashed line marks the real cosmic-web value. z-scores indicate how many standard deviations the real value lies from the null mean.*

### 4.9 Replication on TNG100-1

To check that our results are not specific to one simulation box, we repeated the analysis on TNG100-1, which uses the same physics as TNG300-1 but in a smaller volume. The smaller box contains fewer high-order streams, so the statistics are noisier, but it still provides a useful independent test.

Table 4 shows that the main results broadly replicate. The bifurcation ratio stays close to the constructal prediction of $R_B = 3$, with $R_B = 2.81$ in TNG100-1 versus 3.05 in TNG300-1. The Hack exponent is also very similar in the two runs, and the mean bifurcation angle is effectively identical at 101.3° in both.

The main difference is the length ratio, which is higher in TNG100-1 (1.61 versus 1.22) and therefore closer to the theoretical prediction of $R_L = 2$. Overall, the cross-simulation comparison supports the idea that the main scaling results are robust and not just an artifact of the TNG300-1 volume.

Table 4: *Cross-simulation comparison: TNG300-1 vs. TNG100-1 at* $z = 0$.

| Metric | TNG300-1 | TNG100-1 | Prediction |
|---|---|---|---|
| $R_B$ | 3.05 | 2.81 | 3 (hex.) |
| $R_L$ | 1.22 | 1.61 | 2.0 |
| Hack $h$ | 0.317 | 0.328 | 0.333 |
| Angle | 101.3° | 101.3° | $\leq 77°$ |

# 5 Discussion

### 5.1 The Cosmic Web as an Optimized Flow Network

Our results paint a largely consistent picture: the cosmic web's filament network exhibits hierarchical scaling laws similar to those that constructal theory predicts for dendritic flow systems. The bifurcation ratio $R_B \approx 3$ matches the hexagonal optimum; the length ratio $R_L \approx 1.5$ falls within the empirical range; Hack's law holds with $h \approx 0.31$ (though slightly below the formal prediction range); and drainage mass scales positively with Strahler order. The bifurcation angle test fails: our 3D derivation shows the constructal optimum is $\leq 77°$, while the measured value is $101°$.

What makes this interesting is that the cosmic web is not shaped by hydrodynamics. There are no channels, no fluid viscosity, and no pressure gradients in the traditional sense. Instead, gravity alone drives matter from voids into filaments and from filaments into clusters. The "current" in this system is the gravitational infall of dark matter and baryons along potential gradients, not viscous pipe flow, yet the resulting geometry is quantitatively similar to a river basin.

Bejan (2022) anticipated this result by arguing that the constructal law is fundamentally about access (the geometric problem of connecting a point to a volume) rather than about any specific transport mechanism. Whether the driving force is gravity, pressure, or an electrochemical gradient, the optimal collecting architecture is the same: a tree with predictable branching ratios.

## 5.2 Hexagonal vs. Square Packing

The observation $R_B \approx 3$ rather than $R_B = 4$ deserves emphasis. As derived in Bejan & Lorente (2008, Ch. 4, §4.17), a network built from hexagonal drainage elements achieves $R_B = 3$ and has *lower global flow resistance* than the $R_B = 4$ square-element alternative. The hexagonal packing is the theoretically optimal configuration.

That the cosmic web preferentially adopts the hexagonal rather than square geometry is consistent with the constructal law's core prediction: flow systems evolve toward configurations of minimum resistance. The cosmic web has had billions of years to "optimize," and our measurements suggest it has converged on the most efficient packing.

## 5.3 Early Equilibrium

The absence of temporal convergence is consistent with an "early equilibrium" interpretation: the constructal architecture was established within the first few billion years and maintained ever since. This is consistent with what we know about cosmic structure formation—the large-scale filamentary skeleton was laid down at $z \sim 2$–$5$ and the topological structure has remained largely frozen since that epoch.

However, as discussed in Section **??**, we cannot rule out the alternative that the scaling ratios are set by initial conditions rather than by optimization. Distinguishing these interpretations is an important open question requiring controlled cosmological simulations.

## 5.4 Three-Dimensional Junction Geometry

Our 3D extension of the constructal angle derivation (Section 2.4) predicts optimal daughter–daughter angles $\leq 77°$ for any multiplicity $N$—*smaller* than the 2D prediction, not larger. The measured $101.3°$ therefore cannot be explained by the constructal framework, even in its 3D form.

We interpret this as evidence that the constructal law operates at different scales with different fidelity. At the *macroscopic* level (network topology, stream hierarchies, drainage scaling) the cosmic web behaves as a constructal system. At the microscopic level of individual junctions, the angle is set by the local physics of gravitational infall, which does not resemble flow through pipes. The measured $101.3°$ may instead reflect the typical angle at which filaments converge toward saddle points in the cosmic density field, a quantity set by the geometry of gravitational collapse rather than by flow-resistance optimization.

This distinction is one of the key takeaways of this study. It suggests that the constructal law applies to the big picture organization of the cosmic web (how streams branch and how mass accumulates), but not to the local details of how filaments meet at individual junctions.

### 5.5 Limitations

Several limitations should be noted. First, while we have replicated key results on TNG100-1 (Section 4.9), both simulations use the same code (AREPO) and cosmological parameters; confirming results with an independent code (e.g., EAGLE) would further strengthen the conclusions. Second, our filament identification relies on DisPerSE, which has its own assumptions about persistence thresholds and smoothing scales. Our sensitivity analysis shows that the results are robust to the pipeline parameters downstream of DisPerSE, but we have not tested different DisPerSE configurations. Third, our "drainage volume" proxy (the smallest convex shape enclosing all nodes in a tree component) is rougher than the well-defined watershed boundaries in river basins; this may contribute to the Hack exponent falling slightly below the predicted range. Finally, our 3D angle derivation assumes symmetric tributaries with pipe-flow resistance scaling; a future derivation incorporating gravitational free-fall dynamics might yield different optimal angles.

## 6 Conclusion

Our key findings are:

1. The bifurcation ratio $R_B \approx 3.05$ matches the hexagonal optimum predicted by constructal theory ($R_B = 3$)—the configuration that minimizes global flow resistance.

2. The length ratio $R_L \approx 1.5$ falls within the empirical range for dendritic networks, though below the theoretical optimum of $R_L = 2$; a 3D derivation is needed to explain the shortfall.

3. Hack's law holds with exponent $h = 0.31$ ($R^2 = 0.88$, $p < 10^{-48}$), slightly below the predicted lower bound of 1/3; the discrepancy may reflect 3D geometry.

4. Drainage mass scales positively with Strahler order (slope = 1.34, $p = 0.0025$), confirming the hierarchical mass accumulation expected in a dendritic flow network.

5. All scaling ratios remain stable from $z = 2.3$ to $z = 0$, consistent with early establishment of the hierarchy, though we cannot rule out that the ratios are set by initial conditions.

6. The bifurcation angle (101.3°) fails the constructal prediction: our 3D derivation shows the optimum is $\leq 77°$ for any multiplicity, indicating that local junction geometry is governed by gravitational physics, not flow-resistance optimization.

7. Results are robust to pipeline parameter choices ($\Delta R_B < 0.015$) and to spanning-tree edge discard.

Of seven quantitative tests, three produce clear support for constructal scaling ($R_B$, width scaling, pipeline robustness), two show partial agreement with caveats ($R_L$, Hack's law), one is ambiguous (temporal stability), and one fails (bifurcation angle). All topological results replicate on an independent simulation volume (TNG100-1).

The pattern of successes and failures is itself revealing. The constructal law accurately predicts the hierarchical topology of the cosmic web, how streams branch, how drainage scales, how mass accumulates, but fails to predict the local junction geometry. This suggests that the constructal law captures the universal geometric logic of connecting a point to a volume, while the fine-grained details depend on the specific physics doing the transporting. In river basins, that physics is viscous flow; in the cosmic web, it is gravitational free-fall. The two produce the same tree structures but different junction angles.

These results support and refine the hypothesis of Bejan (2022): the cosmic web is a constructal system at the topological level, shaped by gravitational tension relief into a dendritic hierarchy that obeys the same macroscopic scaling laws as river basins and vascular networks.

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